Volume of Shale and Clay analysis in dual porosity systems

White Paper

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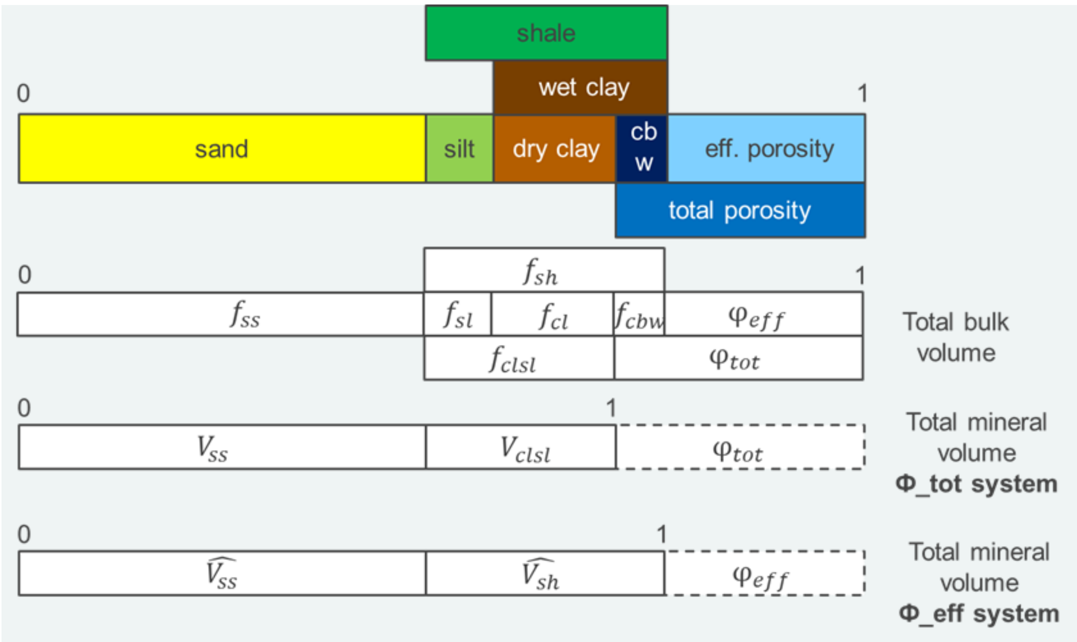

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1.1 Introduction

The objective of this document is to discuss the two fundamentally different approaches to describe shaly sand systems based on the so-called dual-water or dual-porosity model. Most often used in the E&P industry is the effective porosity approach, where all non-sand components such as silt, clay and clay-bound water are lumped together as shale. Hence, in this system we have a volume of sand and a volume of shale log together with effective porosity. The alternative procedure to handle shaly sand systems is to combine the effective porosity with the clay-bound water volume to the total porosity. The mineral volume part is then set-up by a volume of sand and a volume of (dry) clay log.

This paper does not discuss which approach is better. There is some practical evidence that the total porosity approach has some advantages. Maybe the Vclay - Total porosity system is better constrained by data and less sensitive to subjective interpretations than the Vshale – Effective porosity system. This will be the topic of a future paper. The main purpose here is to present the mathematical derivation of the transformation of volume fraction logs between the two different porosity systems. This is not straightforward as one might think at first sight.

Modern rock physics software such as RokDoc 1D/2D from company IkonScience provide tools to generate these different volume fraction logs and to convert data from one to the other porosity system. The starting point of this work actually was a new volume fraction generator tool in RokDoc, which requires more input and delivers more output as an earlier version. At the beginning, the functionality of this new tool was not very clear.

1.2 Volume fraction generator

The Volume Fraction Generator in RokDoc has got a new functionality compared to previous releases. The tool computes, as output logs, the volume of clay, volume of shale and clay-bound water. Necessary input is, as in former versions, a source log, which in most cases should be a gamma ray log. But in addition, the density of (dry) clay and shale must be given. At first sight, this unconventional approach is a bit confusing. In the volume fraction generator window, the two red lines on the gamma ray (or source) log track can be moved interactively so that the left (minimum) line represents the sand line, the right (maximum) line represents the clay/shale line. These values can also be typed in on the right panel.

The application proceeds in three steps. First, a Volume of clay/silt log is determined from the gamma ray (or source) log value (GR_log) and the two chosen extreme gamma ray values of pure sand (GR_sand) and pure clay (GR_clay)

$$V_{clsl} = \frac{GR_{\log} - GR_{sand}}{GR_{clay} - GR_{sand}}.$$

Volume of clay is a linear function of the gamma ray log between the sand and shale lines. Secondly, the porosity of shale is computed from the two input densities of clay and shale (symbols will be explained below in the derivation section)

$$\tilde{\varphi}_{sh} = \frac{\rho_{clsl} - \rho_{sh}}{\rho_{clsl} - \rho_{cbw}} .$$

At last, the Volume of shale (as part of an effective porosity system) is computed by

$$\hat{V}_{sh} = \frac{V_{clsl}}{\left(1 - \tilde{\varphi}_{sh} + \tilde{\varphi}_{sh} V_{clsl}\right)} .$$

Usually in a petrophysical evaluation, the porosity of shale is specified somehow. Porosity of shale is a poorly defined quantity. In RokDoc, the porosity of shale is computed from the densities of clay and shale, which postpones the difficult estimation step from "shale porosity" to "shale density". Dry clay has a density of 2.68 g/cc. The densities of other clay minerals do not vary much, for example illite has 2.71 g/cc. However, kaolinite is a bit lighter. The main problem is to get a reasonable density value for shale. The default value in RokDoc is 2.35 g/cc. This leads to a rather high shale porosity of 19.6%. Usually assumed shale porosity in petrophysical applications is in the 10-15 % range.

1.3 Porosity systems

The volume logs of clay and shale are closely related to the porosity system used. The volume of (dry) clay is related to the total porosity system, because the clay-bound water is added to the effective (free) porosity. The volume of shale, which comprises silt, dry clay and clay-bound water, is related to the effective porosity system (Figure 1).

Many rock physical applications depend on the porosity system. The Fluid substitution is an example. The first input in RokDoc's fluid substitution window is the information whether we deal with an effective or a total porosity system. The first cut-off (at the bottom) refers to either the Volume of shale (effective porosity) or Volume of clay (total porosity). In the project settings, it is possible to set the default porosity system. The default is effective porosity, as this is the most often used one in industry.

In RokDoc there are several conversion tools, which can be found under *Well ops -> Petrophysics -> Porosity System Conversions*. There are tools to convert porosity logs from total to effective or vice versa. For the transformation we need 2 of the following 3 items; Volume of clay log, Volume of shale log, or porosity of shale (via the two densities).

Volume fraction sets can also be converted. Here we need to specify the two densities for (dry) clay and shale. The saturation sets must be converted, too. For this conversion we need to input both the effective as well as the total porosity logs.

Quite often, petrophysicists derive a volume of shale log directly from the gamma ray log. By doing this, it is implicitly assumed that the (non-radiating) silt content and the clay-bound water have always the same relative abundance with respect to the (radiating) dry clay content. Only in this case, the volume of shale is linearly proportional to the gamma ray. To do this with the new Volume fraction generator in RokDoc, we have to give the same values for both the clay and shale density (yielding zero shale porosity). Then, we save only the Volume of shale log.

In case we want to perform our own porosity and volumes calculations or check an existing petrophysical analysis, which used a certain shale porosity value, we could calculate the corresponding shale density from shale porosity

$$\rho_{sh} = \rho_{clsl} - (\rho_{clsl} - 1) \cdot \tilde{\varphi}_{sh} \text{ , or}$$

$$\rho_{sh} = 2.68 - 1.68 \cdot \tilde{\varphi}_{sh} .$$

This shale density, together with the unchanged clay density of 2.68 g/cc, will result in the same shale porosity.

1.4 Derivation of formulas

In this section we want to derive the corresponding formulas. Figure 1 (upper part) shows a diagram illustrating the different ways how we can put the various lithology constituents of a shaly sand system together. This is sometimes called dual-water model or dual porosity model.

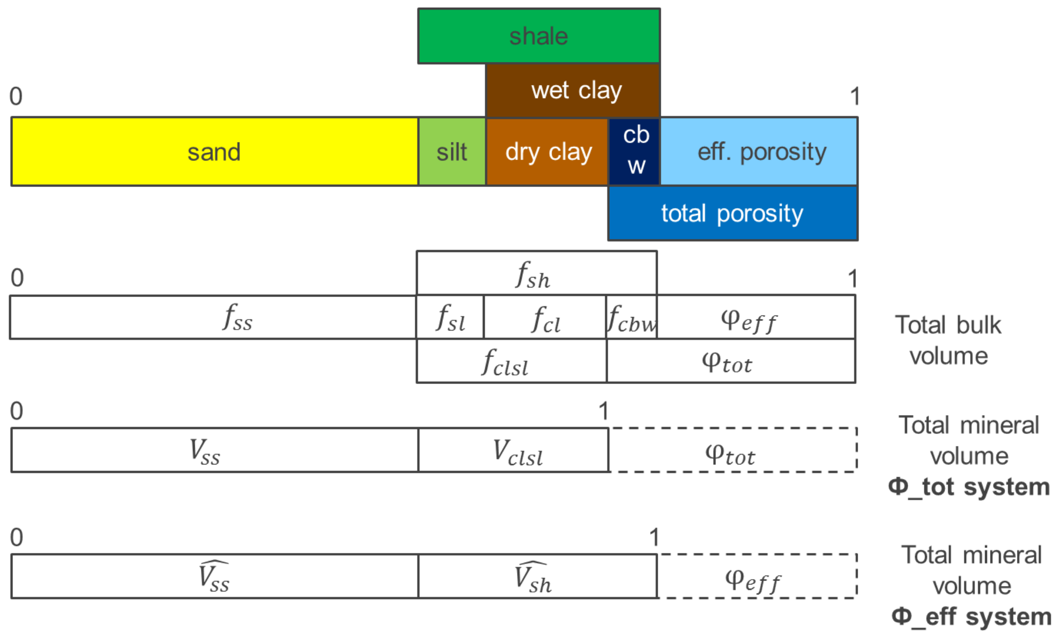

Fig. 1. *Effective and total porosity in shaly sand systems (dual-water model)*

The dry clay always comes with the clay-bound water. The clay together with the water is sometimes called wet clay. The shale is formed by combining the wet clay and the silt fraction. The effective porosity is the free pore space of the rock, excluding the clay-bound water. The total porosity is the sum of effective porosity and the volume of the clay-bound water.

At the lower part of Figure 1 the different symbols of all used quantities in the derivation are given. The small letters f_i and φ_i are the volume fractions of the total bulk volume, adding up to 1. The capital letters V_i designate the mineral volume fractions of a total porosity system (sand and clay-silt). To emphasize the difference between the porosity systems we use the capital letters $\hat{V}_i$ to designate the mineral volume fractions of an effective porosity system (sand and shale). The subscripts have the following meaning: ss = sand, sh = shale, cl = (dry) clay, sl = silt, clsl = clay-silt, cbw = clay-bound water, wcl = wet clay, eff = effective, tot = total.

For the two porosity systems we first write the basic equations defining the relations between the various parts.

Total porosity system	**Effective porosity system**
$V_{ss} + V_{sl} + V_{cl} = 1$	$\hat{V}_{ss} + \hat{V}_{sl} + \hat{V}_{wcl} = 1$
$V_{ss} + V_{clsl} = 1 \quad , \quad V_{sl} + V_{cl} = V_{clsl}$	$\hat{V}_{ss} + \hat{V}_{sh} = 1 \quad , \quad \hat{V}_{sl} + \hat{V}_{wcl} = \hat{V}_{sh}$
$f_{ss} + f_{clsl} + \varphi_{tot} = 1$	$f_{ss} + f_{sh} + \varphi_{eff} = 1$
$\frac{f_{ss}}{1-\varphi_{tot}} + \frac{f_{clsl}}{1-\varphi_{tot}} = 1$	$\frac{f_{ss}}{1-\varphi_{eff}} + \frac{f_{sh}}{1-\varphi_{eff}} = 1$
$V_{ss} = \frac{f_{ss}}{1-\varphi_{tot}}$ (1)	$\hat{V}_{ss} = \frac{f_{ss}}{1-\varphi_{eff}}$ (2)
$V_{clsl} = \frac{f_{clsl}}{1-\varphi_{tot}} = \frac{f_{sl} + f_{cl}}{1-\varphi_{tot}}$ (3)	$\hat{V}_{sh} = \frac{f_{sh}}{1-\varphi_{eff}}$ (4)
$V_{sl} = \frac{f_{sl}}{1-\varphi_{tot}}$	$\hat{V}_{sl} = \frac{f_{sl}}{1-\varphi_{eff}}$
$V_{cl} = \frac{f_{cl}}{1-\varphi_{tot}}$	$\hat{V}_{wcl} = \frac{f_{wcl}}{1-\varphi_{eff}}$

Equations (1) to (4) are found by comparing the relations in the second and fourth row.

1.4.1 Derivation of shale porosity

First we derive the equation for the shale porosity, using the two densities of shale and clay.

We start with the mass balancing equation, the shale being composed of dry clay, silt and clay-bound water

$$\rho_{sh} f_{sh} = \rho_{cl} f_{cl} + \rho_{sl} f_{sl} + \rho_{cbw} f_{cbw} . \qquad (5)$$

The shale porosity $\tilde{\varphi}_{sh}$ is defined as the ratio of volume of clay-bound water to the entire volume of shale. We denote this shale porosity as well as other volume fractions with respect to volume of shale by a tilde above the symbol. The shale porosity is

$$\tilde{\varphi}_{sh} = \tilde{f}_{cbw} = \frac{f_{cbw}}{f_{sh}} = \frac{f_{cbw}}{f_{cl} + f_{sl} + f_{cbw}} \qquad (6) \text{ and } \quad 1 - \tilde{\varphi}_{sh} = \frac{f_{cl} + f_{sl}}{f_{sh}} = \tilde{f}_{cl} + \tilde{f}_{sl} .$$

In the following derivation we lump together the (non-radiating) silt and the (radiating) clay and use the subscript "clsl" for the clay-silt fraction. First we divide equation (5) by f_{sh}

$$\rho_{sh} = \rho_{cl} \tilde{f}_{cl} + \rho_{sl} \tilde{f}_{sl} + \rho_{cbw} \tilde{\varphi}_{sh} . \qquad (7)$$

With the newly defined density of the combined clay-silt fraction

$$\boxed{\rho_{clsl} = \rho_{cl} \frac{\tilde{f}_{cl}}{\tilde{f}_{cl} + \tilde{f}_{sl}} + \rho_{sl} \frac{\tilde{f}_{sl}}{\tilde{f}_{cl} + \tilde{f}_{sl}} ,} \qquad (8)$$

we write equation (7) as

$$\rho_{sh} = \rho_{clsl} (1 - \tilde{\varphi}_{sh}) + \rho_{cbw} \tilde{\varphi}_{sh} .$$

From this we get the shale porosity

$$\boxed{\tilde{\varphi}_{sh} = \frac{\rho_{clsl} - \rho_{sh}}{\rho_{clsl} - \rho_{cbw}} .} \qquad (9)$$

1.4.2 Derivation of Vclay-to-Vshale relationship

We start with the defining equation (6) of shale porosity and write (see also Figure 1)

$$f_{cbw} = \tilde{\varphi}_{sh} f_{sh} = f_{sh} - f_{clsl} ,$$

or rearranging

$$f_{clsl} = f_{sh} (1 - \tilde{\varphi}_{sh}) .$$

Using the equations (3-4) we obtain the following relationship between volume of clay and volume of shale, using the three porosities $\varphi_{tot}, \varphi_{eff}$ and $\tilde{\varphi}_{sh}$,

$$V_{clsl} = \hat{V}_{sh} \frac{(1-\tilde{\varphi}_{sh})(1-\varphi_{eff})}{(1-\varphi_{tot})}. \qquad (10)$$

Next we derive a relationship between these three porosities,

$$\varphi_{tot} = \varphi_{eff} + f_{cbw},$$

$$1-\varphi_{tot} = 1-\varphi_{eff} - f_{cbw},$$

$$1-\varphi_{tot} = 1-\varphi_{eff} - f_{sh}\tilde{\varphi}_{sh},$$

$$1-\varphi_{tot} = 1-\varphi_{eff} - (1-\varphi_{eff})\hat{V}_{sh}\tilde{\varphi}_{sh},$$

$$1-\varphi_{tot} = (1-\varphi_{eff})(1-\hat{V}_{sh}\tilde{\varphi}_{sh}).$$

Using the last equation we can eliminate two porosities (effective and total) from equation (10) and write

$$\boxed{V_{clsl} = \hat{V}_{sh} \frac{(1-\tilde{\varphi}_{sh})}{(1-\hat{V}_{sh}\tilde{\varphi}_{sh})}.} \qquad (11)$$

Inverting this relationship we obtain

$$\boxed{\hat{V}_{sh} = \frac{V_{clsl}}{(1-\tilde{\varphi}_{sh}+\tilde{\varphi}_{sh}V_{clsl})}.} \qquad (12)$$

Solving for the porosity of shale

$$\boxed{\tilde{\varphi}_{sh} = \frac{(\hat{V}_{sh}-V_{clsl})}{\hat{V}_{sh}(1-V_{clsl})}.} \qquad (13)$$

For later use we write also the expression

$$1-\tilde{\varphi}_{sh} = \frac{V_{clsl}(1-\hat{V}_{sh})}{\hat{V}_{sh}(1-V_{clsl})}. \qquad (14)$$

The Vshale to Vclay relationship (12) is plotted for several constant values of shale porosity (Figure 2). All curves are nonlinear; Vshale is always larger than Vclay. In the unrealistic case of vanishing shale porosity, Vshale is equal to Vclay. In the unrealistic case of 100% shale porosity, Vshale is equal to 100%. If there is no clay, there is no shale, either. If there is 100% clay, there is of course also 100% shale, independent of shale porosity.

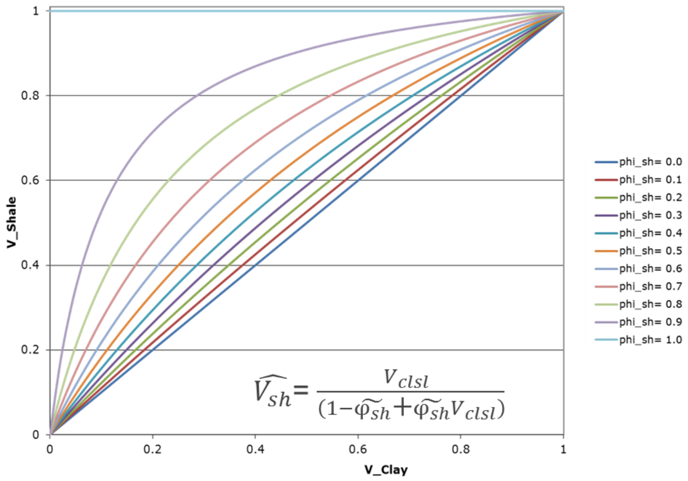

Fig. 2. *Vshale to Vclay relationship (eq. (12))*

1.4.3 Derivation of porosity conversion relationships

We rewrite equation (10) as

$$V_{clsl}(1-\varphi_{tot}) = \hat{V}_{sh}(1-\tilde{\varphi}_{sh})(1-\varphi_{eff}). \qquad (15)$$

From this relationship we see that we can convert from total to effective porosity (or vice versa) if we know two of the three quantities; clay volume, shale volume or shale porosity. The third quantity can be computed from the two others (eq. (11-13)).

We can rewrite equation (15) as

$$\frac{(1-\varphi_{tot})}{(1-\varphi_{eff})} = a \qquad (16)$$

with

$$a = \frac{\hat{V}_{sh}(1-\tilde{\varphi}_{sh})}{V_{clsl}}.$$

Using eq. (11-13) we get three different expressions for the quantity a

$$a = 1-\hat{V}_{sh}\tilde{\varphi}_{sh}, \qquad (17)$$

$$a = \frac{(1-\tilde{\varphi}_{sh})}{(1-\tilde{\varphi}_{sh}(1-V_{clsl}))}, \qquad (18)$$

$$a = \frac{(1-\hat{V}_{sh})}{(1-V_{clsl})}. \qquad (19)$$

Then the total porosity is

$$\varphi_{tot} = 1 - a\left(1 - \varphi_{eff}\right). \tag{20}$$

The effective porosity is

$$\varphi_{eff} = 1 - \frac{1}{a}\left(1 - \varphi_{tot}\right). \tag{21}$$

These formulas are used in the porosity conversion tool in RokDoc.

1.4.4 Derivation of volume fraction relationship

We start from equations (1-2) and obtain for example for the sand fraction (or any other lithology component except shale or clay)

$$f_{ss} = V_{ss}\left(1 - \varphi_{tot}\right) = \hat{V}_{ss}\left(1 - \varphi_{eff}\right)$$

or

$$V_{ss} = \hat{V}_{ss}\frac{\left(1 - \varphi_{eff}\right)}{\left(1 - \varphi_{tot}\right)}.$$

Using equation (16-17) we obtain the relationship to get the volume fractions of the total porosity system

$$V_{ss} = \hat{V}_{ss}\frac{1}{\left(1 - \hat{V}_{sh}\tilde{\varphi}_{sh}\right)}. \tag{22}$$

The volume fractions of the effective porosity system are calculated from those of the total system as

$$\hat{V}_{ss} = V_{ss}\left(1 - \hat{V}_{sh}\tilde{\varphi}_{sh}\right).$$

But because shale volume might not be known or available we need to substitute equation (12) or use eq. (17-18). Finally, we get the effective volume fraction as function of total volume fractions and shale porosity

$$\hat{V}_{ss} = V_{ss}\frac{\left(1 - \tilde{\varphi}_{sh}\right)}{\left(1 - \tilde{\varphi}_{sh}\left(1 - V_{clsl}\right)\right)}. \tag{23}$$

These formulas are used in the volume fraction conversion tool in RokDoc, where one needs to specify the two densities to get the shale porosity.